# CONDUCTORS, TEST IDEALS, AND IDEALS GENERATED BY SYSTEMS OF PARAMETERS

KAITO KIMURA

The main focus of this talk is on the following two questions.

**Question** (1) Isn't the conductor of an analytically unramified local ring contained in every ideal generated by a system of parameters?
(2) Isn't the parameter test ideal of a local ring of prime characteristic contained in every ideal generated by a system of parameters?

Ikeda [5] showed that the monomial conjecture holds precisely when Question (1) is affirmative for all Gorenstein complete local domains, and asked which domains satisfy this. Since the monomial conjecture has been resolved affirmatively by André [1], Question (1) holds for all Gorenstein complete local domains. Asgharzadeh [2] proved that Question (1) is affirmative for all analytically unramified quasi-Gorenstein local rings. In [4], Huneke and Swanson explicitly posed the open problem of whether Question (1) holds for all analytically unramified Cohen–Macaulay local domains.

For a reduced local ring of prime characteristic, the conductor contains the parameter test ideal, and hence if Question (2) is answered affirmatively, then so is Question (1). Smith [7] showed that Question (2) (and thus (1)) holds for any Gorenstein reduced complete local ring of prime characteristic. Dey and Dutta [3] proved that the parameter test ideal of a Gorenstein complete local domain is not contained in any ideal of finite projective dimension, and in particular, not in any ideal generated by a system of parameters.

The aim of this talk is to make progress on these questions, building on the aforementioned previous studies. This talk is based on a preprint [6].

## References

[1] Y. André, La conjecture du facteur direct, *Publ. Math. Inst. Hautes Études Sci.* **127** (2018), 71–93.
[2] M. Asgharzadeh, Homology of conductor, preprint (2024), `arXiv:2404.02005`.
[3] S. Dey; M. Dutta, On containment of trace ideals in ideals of finite homological dimension, preprint (2024), `arXiv:2312.01230`.
[4] C. Huneke; I. Swanson, Integral closure of ideals, rings, and modules, *London Math. Soc. Lecture Note Ser.*, **336** *Cambridge University Press, Cambridge*, (2006).
[5] S. Ikeda, Conductor ideals of Gorenstein domains and local rings of multiplicity 2, *The Annals of Gifu University for Education and Languages* **33** (1997) 109–116.
[6] K. Kimura, Trace ideals, conductors, and ideals of finite (phantom) projective dimension, preprint (2025), `arXiv:2501.03442`.
[7] K. E. Smith, Test ideals in local rings, *Trans. Amer. Math. Soc.* **347** (1995), no. 9, 3453–3472.

Graduate School of Mathematics, Nagoya University, Furocho, Chikusaku, Nagoya 464-8602, Japan
*Email address*: `m21018b@math.nagoya-u.ac.jp`